SiC; 3) the carbide was formed but was subsequently consumed by secondary reactions yielding gaseous products; and, last but not least, 4) the SiC escaped detection because of analytical difficulties. The Note discussed the chemically interesting cases 1 and 2, and provided experimental evidence favoring case 3. The main purpose of the Note was to demonstrate that the mere absence of SiC in chars cannot be taken as a proof of the nonparticipation of the char-reinforcement reactions.

That the solid-solid reactions do occur in ablators is illustrated by the case of zirconia quoted in the Comment, the fact that carborundum is obtained from carbon and sand at comparable temperatures, and the fact that SiC does appear in some chars. The significance of the C-SiO_2 reactions may range from nil to high, depending on temperature and pressure; at relatively moderate conditions ("orderly" ablation) it will certainly be lower than that of the more facile reactions of carbon with volatiles (H_2O, CO_2).

Both the carbon-silica and the carbon-volatiles reactions consume carbon and deplete the protective char layer, hence are detrimental; they bring, however, some thermal compensation. The loss of a carbon atom from char via the interaction with silica is better compensated than the loss incurred by reactions with volatiles (and, of course, by shear forces and spallation):

$$C + CO_2 \rightarrow 2\,CO \qquad \Delta H_{1600} = 39.2 \text{ kcal/mole} \quad (1)$$

$$C + H_2O \rightarrow CO + H_2 \qquad \Delta H_{1600} = 33.2 \text{ kcal/mole} \quad (2)$$

$$C + SiO_2 \rightarrow CO + SiO \qquad \Delta H_{1600} = 160.6 \text{ kcal/mole} \quad (3)$$

Reaction (3) consumes also the silica, resulting in problems discussed in the Comment. In the case of the re-entry heat shields, where some material loss can be tolerated, the high endothermicity of the C-SiO_2 reactions (along with that of the dissociative vaporization of silica) is welcome; it does not "account for the heat of ablation," but it may contribute to it significantly. But in cases where dimensional stability is critical, such as rocket nozzles, no heat effect gained can adequately compensate for the damage done. All material-consuming processes are undesirable. Nevertheless, the surface or near-surface potential interactions have to be studied, if only to avoid them by judicious choice of an "ablative" material.

The Note dealt specifically with the chemistry of the carbon-silica system. Such aspects as structural integrity of ablators, effect of shear forces, effect of external gases, etc., were outside the scope of the Note. The Comment draws attention to these important aspects and to their practical implications. Such practical matters as the form of reinforcement, i.e., whether a chopped fabric or a shingle-oriented wrapped tape is used, may spell a large difference in actual performance. But it is not merely academic to know the reasons for the limitations of materials, if their methodical improvement is contemplated.

Chemical reactions in ablation are often regarded as a source of trouble. Indeed, they consume material, are sometimes exothermic, and altogether bring excessive variability to the orderly system. An engineer's dream is to have a super-material that would serve the purpose and remain itself unchanged. Ablation is then a poor substitute, but the only effective substitute available, at least for re-entry. This attitude was discussed by Gruntfest and Shenker.[2] Until this super-material is found, chemical reactions with all their benefits and disadvantages will have to be contended with.

References

[1] Ladacki, M., "Silicon carbide in ablative chars," AIAA J. 4, 1445–1447 (1966).

[2] Gruntfest, I. J. and Shenker, L. H., "Ablation," *Modern Science and Technology*, edited by R. Colborn (D. Van Nostrand Company Inc., Princeton, N. J., 1965), p. 535.

Comment on "Duct-Flow of Conducting Fluids under Arbitrarily Oriented Applied Magnetic Fields"

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In Ref. 1, the author considers a generalized transformation to uncouple the system of magnetic and fluid velocity variables. By this transformation he uncouples the system, introducing two variables f and g in place of the magnetic and fluid velocity components h and v in the flow direction, defined in Eqs. (23) of Ref. 1, and arrives at Eqs. (24) for f, g, which satisfy identical equations. Transforming the boundary conditions as well, he finally arrives at the governing equations for f, g together with boundary conditions on f, g, given in Eqs. (27).

The object of this note is to give a slightly different transformation, which uncouples the system; simplifies the boundary conditions on f, g; and finally, contributes to the simplicity of the solutions. Formulas (30) and (31)[1] can now be replaced by simpler ones from which the solution in the nonmagnetic case is obtained directly by simple considerations. The same transformation applied to the single-series solution gives a much simpler result and from this, the case of Shercliff[2] can be obtained directly. The case of fully developed flow in the nonmagnetic case cannot be derived from either the double-series or single-series solutions of Eraslan, the difficulty arising from the type of generalized transformation used in Eqs. (23). Further, the single-series solutions of h and v given in Eqs. (37) and (38) do not converge because of the presence of a term

$$[\gamma_2 C_4/(\omega^2 - \alpha^2)]ye^{-\alpha y}$$

in the expansion of $g(m, y)$ as in (35). C_4 should read

$$\frac{m\pi}{M(\gamma_1^2 + \gamma_2^2)}\,[1 + (-1)^{m+1}e^{-(M/2)\gamma_1}]$$

The new transformation given in this note overcomes the difficulty of convergence of the single series for h and v and also gives Shercliff's result[2] as a particular case.

Transformation for Uncoupling the System

Consider the generalized transformations

$$h(x, y) = \tfrac{1}{2}[e^{-(M/2)(\gamma_1 x+\gamma_2 y)}f(x, y) + e^{(M/2)(\gamma_1 x+\gamma_2 x)}g(x, y)] \quad (1)$$

$$v(x, y) = \tfrac{1}{2}[e^{-(M/2)(\gamma_1 x+\gamma_2 y)}f(x, y) - e^{(M/2)(\gamma_1 x+\gamma_2 y)}g(x, y)]$$

with the same boundary conditions as given in Eqs. (26) of Ref. 1. They are

$$\begin{aligned} h(0, y) &= 0 & v(0, y) &= 0 \\ h(\mu, y) &= 0 & v(\mu, y) &= 0 \\ h(x, 0) &= 0 & v(x, 0) &= 0 \\ h(x, \lambda) &= 0 & v(x, \lambda) &= 0 \end{aligned} \quad (2)$$

The governing equations for f and g are:

$$\nabla^2 f - \frac{M^2}{4}(\gamma_1^2 + \gamma_2^2)f = -e^{(M/2)(\gamma_1 x+\gamma_2 y)} \quad (3)$$

$$\nabla^2 g - \frac{M^2}{4}(\gamma_1^2 + \gamma_2^2)g = e^{-(M/2)(\gamma_1 x+\gamma_2 y)} \quad (4)$$

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The boundary conditions on f and g are:

$$f(0, y) = 0 \qquad f(x, 0) = 0 \qquad f(\mu, y) = 0 \qquad f(x, \lambda) = 0 \tag{5}$$

$$g(0, y) = 0 \qquad g(x, 0) = 0 \qquad g(\mu, y) = 0 \qquad g(x, \lambda) = 0 \tag{6}$$

Defining two double, finite Fourier sine transformations of the form (μ is introduced in order to compare the result with that of Shercliff,[2] where the origin is taken at the center),

$$F(m, n) = \int_0^\lambda \int_0^\mu f(x, y) \sin\frac{m\pi x}{\mu} \sin\frac{n\pi y}{\lambda}\, dxdy \tag{7}$$

$$G(m, n) = \int_0^\lambda \int_0^\mu g(x, y) \sin\frac{m\pi x}{\mu} \sin\frac{n\pi y}{\lambda}\, dxdy \tag{8}$$

with the inversion

$$f(x, y) = \frac{4}{\lambda\mu} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F(m, n) \sin\frac{m\pi x}{\mu} \sin\frac{n\pi y}{\lambda} \tag{9}$$

$$g(x, y) = \frac{4}{\lambda\mu} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} G(m, n) \sin\frac{m\pi x}{\mu} \sin\frac{n\pi y}{\lambda} \tag{10}$$

and applying the transformations (7), (8), to Eqs. (3) and (4) and making use of (5) and (6), one gets

$$F(m, n) = \frac{64mn\pi^2\lambda^3\mu^3[1 + (-1)^{m+1}e^{(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{(M/2)\gamma_2\lambda}]}{[M^2(\gamma_1^2 + \gamma_2^2)\lambda^2\mu^2 + 4\pi^2(m^2\lambda^2 + n^2\mu^2)][M^2\gamma_1^2\mu^2 + 4m^2\pi^2][M^2\gamma^2\lambda^2 + 4n^2\pi^2]} \tag{11}$$

$$G(m, n) = \frac{-64mn\pi^2\lambda^3\mu^3[1 + (-1)^{m+1}e^{-(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{-(M/2)\gamma_2\lambda}]}{[M^2(\gamma_1^2 + \gamma_2^2)\lambda^2\mu^2 + 4\pi^2(m^2\lambda^2 + n^2\mu^2)][M^2\gamma_1^2\mu^2 + 4m^2\pi^2][M^2\gamma_2^2\lambda^2 + 4n^2\pi^2]} \tag{12}$$

From these equations the nonmagnetic case of a fully developed viscous flow in a rectangular duct, when the velocity of the fluid is parallel to the axis of the duct, can be obtained directly by taking $M = 0$ or by taking $\gamma_1 = 0 = \gamma_2$. In the nonmagnetic case $v = f, h = 0, g = 0$, so that

$$v = \sum_{m,\, n \text{ odd}} \sum \frac{16\lambda^2\mu^2 \sin(m\pi x/\mu) \sin(n\pi y/\lambda)}{\pi^4 mn(m^2\lambda^2 + n^2\mu^2)} \tag{13}$$

With the help of Eqs. (9–12), one writes

$$h(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{128\pi^2\lambda^2\mu^2 mn \sin(m\pi x/\mu) \sin(n\pi y/\lambda)}{[M(\gamma_1^2 + \gamma_2^2)\lambda^2\mu^2 + 4\pi^2(m^2\lambda^2 + n^2\mu^2)][M^2\gamma_1^2\mu^2 + 4m^2\pi^2][M^2\gamma_2^2\lambda^2 + 4n^2\pi^2]} \times \{[1 + (-1)^{m+1} e^{(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{(M/2)\gamma_2\lambda}]e^{-(M/2)(\gamma_1 z + \gamma_2 y)} - [1 + (-1)^{m+1}e^{-(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{-(M/2)\gamma_2\lambda}]e^{(M/2)(\gamma_1 x + \gamma_2 y)}\} \tag{14}$$

$$v(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{128\pi^2\lambda^2\mu^2 mn \sin(m\pi x/\mu) \sin(m\pi y/\lambda)}{[M^2(\gamma_1^2 + \gamma_2^2)\lambda^2\mu^2 + 4\pi^2(m^2\lambda^2 + n^2\mu^2)][M^2\gamma_1^2\mu^2 + 4m^2\pi^2][M^2\gamma_2^2\lambda^2 + 4m^2\pi^2]} \times \{[1 + (-1)^{m+1}e^{(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{(M/2)\gamma_2\lambda}]e^{-(M/2)(\gamma_1 x + \gamma_2 y)} + [1 + (-1)^{m+1}e^{-(M/2)\gamma_1\mu}][1 + (-1)^{n+1}e^{-(M/2)\gamma_2\lambda}]\, e^{(M/2)(\gamma_1 x + \gamma_2 y)}\} \tag{15}$$

Defining two finite Fourier sine transformations of the form

$$F(m, y) = \int_0^\mu f(x, y) \sin\frac{m\pi x}{\mu}\, dx \tag{16}$$

$$G(m, y) = \int_0^\mu g(x, y) \sin\frac{m\pi x}{\mu}\, dx \tag{17}$$

with the inversion

$$f(x, y) = \frac{2}{\mu} \sum_{m=1}^{\infty} F(m, y) \sin\frac{m\pi x}{\mu} \tag{18}$$

$$g(x, y) = \frac{2}{\mu} \sum_{m=1}^{\infty} G(m, y) \sin\frac{m\pi x}{\mu} \tag{19}$$

and applying transformations (16) and (17) to Eqs. (3) and(4), one gets

$$F(m, y) = \frac{16m\pi\mu^3[1 + (-1)^{m+1}e^{(M/2)\gamma_1\mu}]}{(M^2\gamma_1^2\mu^2 + 4m^2\pi^2)^2} \times \left\{e^{(M/2)\gamma_2 y} - \frac{e^{\omega y}(e^{(M/2)\gamma_2\lambda} - e^{-\omega\lambda}) - e^{-\omega y}(e^{(M/2)\gamma_2\lambda} - e^{\omega\lambda})}{2 \sinh\omega\lambda}\right\} \tag{20}$$

$$G(m, y) = \frac{-16m\pi\mu^3[1 + (-1)^{m+1}e^{-(M/2)\gamma_1\mu}]}{(M^2\gamma_1^2\mu^2 + 4m^2\pi^2)^2}\left\{e^{-(M/2)y_2\gamma} - \frac{e^{\omega y}(e^{-(M/2)\gamma_2\lambda} - e^{-\omega\lambda}) - e^{-\omega y}(e^{-(M/2)\gamma_2\lambda} - e^{\omega y})}{2 \sinh\omega\lambda}\right\} \tag{21}$$

where

$$\omega^2 = (M^2/4)(\gamma_1^2 + \gamma_2^2) + (m^2\pi^2/\mu^2)$$

One can write

$$h + v = 32\pi\mu^2 e^{-(M/2)\gamma_1 x} \sum_{m=1}^{\infty} \frac{m \sin(m\pi x/\mu)[1 + (-1)^{m+1} e^{(M/2)\gamma_1 \mu}]}{(M^2\gamma_1^2\mu^2 + 4m^2\pi^2)^2} \times \left\{1 - \frac{e^{\omega y - (M/2)\gamma_2 y}(e^{(M/2)\gamma_2\lambda} - e^{-\omega\lambda}) - e^{-\omega y - (M/2)\gamma_2 y}(e^{(M/2)\gamma_2\lambda} - e^{\omega\lambda})}{2 \sinh \omega\lambda}\right\} \tag{22}$$

Shercliff's[2] result follows from (22) by taking

$$\gamma_1 = 0 \qquad \gamma_2 = 1 \qquad \mu = 2l \qquad \lambda = 2$$

Shercliff's[2] result can be put in the form

$$h + v = \frac{2}{l} \sum_{K=0}^{\infty} \frac{(-1)^k \cos\alpha_K(x - l)}{\alpha_K^3} \times \left\{1 - \frac{e^{\gamma K(y-1)} \sinh\beta_K + e^{-\beta K(y-1)} \sinh\gamma_K}{\sinh(\beta_K + \gamma_K)}\right\} \tag{23}$$

where

$$\alpha_K = (K + \tfrac{1}{2})(\pi/l)$$

$$\beta_K = \tfrac{1}{2}[M + (M^2 + 4\alpha_K^2)^{1/2}]$$

$$\gamma_K = \tfrac{1}{2}[-M + (M^2 + 4\alpha_K^2)^{1/2}]$$

Shifting the origin from the corner of the cross section of the duct to its center by means of

$$x - l = \xi \qquad y - 1 = \eta$$

$$h + v = \frac{2}{l} \sum_{K=0}^{\infty} \frac{(-1)^K \cos\alpha_K\xi}{\alpha_K^3} \times \left[1 - \frac{e^{\gamma_K\eta} \sinh\beta_k + e^{-\beta_K\eta} \sinh\gamma_K}{\sinh(\beta_K + \gamma_K)}\right] \tag{24}$$

For $M = 0$, the solution (22) reduces to

$$v = \frac{4\mu^2}{\pi^3} \sum_{K=0}^{\infty} \frac{(-1)^K \cos(2K+1)(\pi\xi/\mu)}{(2K + 1)^3} \times \left[1 - \frac{\cosh(2K + 1)(\pi\eta/\mu)}{\cosh(2K + 1)(\pi\lambda/2\mu)}\right] \tag{25}$$

where the origin is shifted to $(\mu/2, \lambda/2)$ by means of

$$x = \xi + (\mu/2) \qquad y = \eta + (\lambda/2)$$

This agrees with the result given in Ref. 3.

References

[1] Eraslan, A. H., "Duct-flow of conducting fluids under arbitrarily oriented applied magnetic fields," AIAA J. **4**, 620–626 (1966).

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[3] Rosenhead, L. (ed.), "Laminar boundary layers," *Fluid Motion Memoir* (Oxford University Press, New York, 1963), p. 136.